Exercise 79

The displacement of a particle on a vibrating string is given by the equation
$s(t) = 10 + \frac{1}{4}\sin(10\pi t)$ where s is measured in centimeters and t in seconds. Find the velocity of
the particle after t seconds.

Solution

The velocity is the derivative of the displacement function.

$$v(t) = \frac{ds}{dt}$$

$$= \frac{d}{dt}[10 + \frac{1}{4}\sin(10\pi t)]$$

$$= \frac{d}{dt}(10) + \frac{1}{4}\frac{d}{dt}[\sin(10\pi t)]$$

$$= (0) + \frac{1}{4}\left[\cos(10\pi t) \cdot \frac{d}{dt}(10\pi t)\right]$$

$$= \frac{1}{4}\left[\cos(10\pi t) \cdot (10\pi)\right]$$

$$= \frac{5\pi}{2}\cos(10\pi t)$$

Since s is in centimeters and t is in seconds, the velocity has units of centimeters per second.